

Exergetic optimization of a thermoacoustic engine using the particle swarm optimization method

Hussein Chaitou*, Philippe Nika

Institut FEMTO-ST/UMR CNRS 6174, Parc Technologique, 2, Avenue Jean Moulin, 90000 Belfort, France

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ABSTRACT

Thermoacoustic engines convert heat energy into acoustic energy. Then, the acoustic energy can be used to pump heat or to generate electricity. It is well-known that the acoustic energy and therefore the exergetic efficiency depend on parameters such as the stack's hydraulic radius, the stack's position in the resonator and the traveling-standing-wave ratio. In this paper, these three parameters are investigated in order to study and analyze the best value of the produced acoustic energy, the exergetic efficiency and the product of the acoustic energy by the exergetic efficiency of a thermoacoustic engine with a parallel-plate stack. The dimensionless expressions of the thermoacoustic equations are derived and calculated. Then, the Particle Swarm Optimization method (PSO) is introduced and used for the first time in the thermoacoustic research. The use of the PSO method and the optimization of the acoustic energy multiplied by the exergetic efficiency are novel contributions to this domain of research. This paper discusses some significant conclusions which are useful for the design of new thermoacoustic engines.

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1. Introduction

The thermoacoustic energy conversion is interpreted as the thermodynamic interaction of a compressible moving oscillating fluid with a solid wall which has a temperature gradient in the direction of the oscillation. During the displacement of the fluid, the variations of the pressure within the fluid generate heat transfers with the wall of the solid, constituting the stack or the regenerator of the thermoacoustic device. According to whether the fluid is heated or cooled during the phases of compression or expansion and according to the direction of its displacement, the fluid carries out a thermodynamic cycle which makes it possible to generate or consume mechanical/acoustic energy. When this energy is generated, the thermoacoustic device is called an engine or a prime mover, and when the energy is consumed, the thermoacoustic device is called a refrigerator or a heat pump. In the case of the thermoacoustic engines, the energy generated can thereafter be used to drive loads such as electrical generators or pulse tube refrigerators.

In fact, the thermoacoustic devices have many advantages. Firstly, these devices can use any external energy sources like solar energy, radio isotope sources, combustion of any fuel oil, biomass, hot gas downstream from cycle, etc. Secondly, these devices do not contain any moving part except the driven loads in the devices coupled to the thermoacoustic systems (for example, the electric

production devices or the actuator of pressure used in the cooler devices). Hence, it is obvious that the thermoacoustic systems offer a high level of reliability and low maintenance requirements. Furthermore, as shown in Fig. 1, these devices basically consist of a stack (when using a standing-wave) and/or a regenerator (when using a traveling-wave), a cold and a hot heat exchanger, and a resonator. That is why the realization cost of these thermoacoustic systems is quite low comparing to other conventional systems. And last, but not least, these devices are also friendly environmental because they use a non-toxic working gas (air, helium, nitrogen, etc.).

The interest to thermoacoustic systems has been arisen and expanded since the early 1980s when Swift from Los Alamos National Laboratory and Garrett from PEN State University, whom can be regarded as the precursors in the field [1,2], have successfully worked and made the first practical thermoacoustic devices that produced a useful work.

Based on the works of Rott [3–9], Swift [10–12] has shown that the boundary functioning between thermoacoustic stack-based engines and heat pumps corresponds to a critical mean temperature gradient across the stack. When the value of the wall temperature gradient is higher than the value of the critical mean temperature gradient, the acoustic wave is created and the thermoacoustic device works as an engine (a prime mover). Moreover, when the value of the wall temperature gradient is less than the value of the critical mean temperature gradient, the acoustic work is consumed and the thermoacoustic device functions as a refrigerator (a heat pump).

* Corresponding author. Tel.: +33 384578220; fax: +33 384570032.

E-mail address: hussein.chaitou@univ-fcomte.fr (H. Chaitou).

Nomenclature

A	cross-sectional area (m^2)
c	speed of local sound (m s^{-1})
c_0	speed of sound (m s^{-1})
c_p	isobaric heat capacity ($\text{J Kg}^{-1} \text{K}^{-1}$)
f	frequency (Hz)
$\dot{H}_x$	total energy flux in the x direction (W)
k_0	wave number (m^{-1})
k	thermal conductivity ($\text{W m}^{-1} \text{K}^{-1}$)
k_{eq}	equivalent thermal conductivity ($\text{W m}^{-1} \text{K}^{-1}$)
L	stack's length (m)
p	pressure (Pa)
Pr	Prandtl number
$\dot{Q}_x$	heat flux in the x direction (W)
r	gas constant ($\text{J Kg}^{-1} \text{K}^{-1}$)
r_h	hydraulic radius (m)
$\dot{S}$	entropy flux (W K^{-1})
T	temperature (K)
t	time (s)
u	velocity (m s^{-1})
$\bar{u}$	volume flow rate ($\text{m}^3 \text{s}^{-1}$)
$\dot{W}_x$	work flux in the x direction (W)
$\langle \rangle$	time-averaged
$\Re[\]$	real part of
$\Im[\]$	imaginary part of

Greek letters

α	thermal expansion coefficient (K^{-1})
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γ	ratio, isobaric to isochoric specific heat
δ	penetration depth (m)
ε	porosity
ε_s	plate heat capacity ratio
ρ	density (kg m^{-3})
λ_0	wave length (m)
η_{ex}	exergetic efficiency
η_{en}	energetic efficiency
μ	dynamic viscosity ($\text{kg m}^{-1} \text{s}^{-1}$)
τ	traveling-standing-wave ratio
ω	angular frequency (rad s^{-1})

Subscripts

0	ambient
a	first order oscillation
g	gas
s	solid
t	thermal
v	viscous

Superscripts

$\bar{f}$	mean, space-averaged perpendicular to x of f
$\dot{f}$	time rate of f
$\bar{f}$	conjugate part of f
f^*	dimensionless number of f

In addition to the two classes of thermoacoustic devices (engines and refrigerators), we can also classify each thermoacoustic devices as a standing-wave or traveling-wave (Fig. 1a and b respectively). In a standing-wave (stack-based devices), the phase between the oscillating velocity of the gas and the oscillating

pressure is close to 90° , and the gas interacts with the stack's wall by a voluntarily imperfect thermal contact. While in a traveling-wave (regenerator-based devices), the phase is close to 0° , and the gas and the regenerator's wall are in very good thermal contact. In 1979, the American engineer Ceperley [13] has discovered that the Stirling machines are not other than traveling-wave thermoacoustic devices. In such a device, energy conversion efficiency can approach the theoretical maximum efficiency, i.e. Carnot efficiency, which depends only on the temperatures of both the hot and the cold thermal reservoirs. The prototypes achieved by Backhaus and Swift [14–18] of “thermoacoustic” Stirling machines have opened the way to the realization of increasingly powerful systems. The mini electric thermoacoustic generator produced by Backhaus et al. [19,20] for NASA offered already an electric output power of 58 W with a total efficiency of 18%, thus approaching the 25% of the efficiency of the gasoline engines with internal combustion (efficiency of the sink to the wheels).

At the present time, thermoacoustic systems, either the engines or the refrigerators, do not reach sufficient exergetic efficiency (the ratio between the system conversion efficiency and the theoretical Carnot efficiency). The most values of output exergetic efficiency usually published are between 10% and 20%, and at best 30%, but an objective of 40% seems however to be realistic if the various sources of energy dissipations or degradations are identified and decreased. For example, in the pulse-tube refrigerators, one can avoid using devices, such as long capillaries, in order to realize the phasing between the pressure and volumetric velocity oscillations, because these are acoustic impedances, which represent a degradation source for the acoustic power. However, one can replace these impedances by adding a feedback acoustical loop to the acoustic network in order to recycle a large part of the acoustic energy carried out from the regenerator back into its entry. But, by doing this, one open the door to other categories of acoustic energy degradations which are named “streaming flows”. However, it is

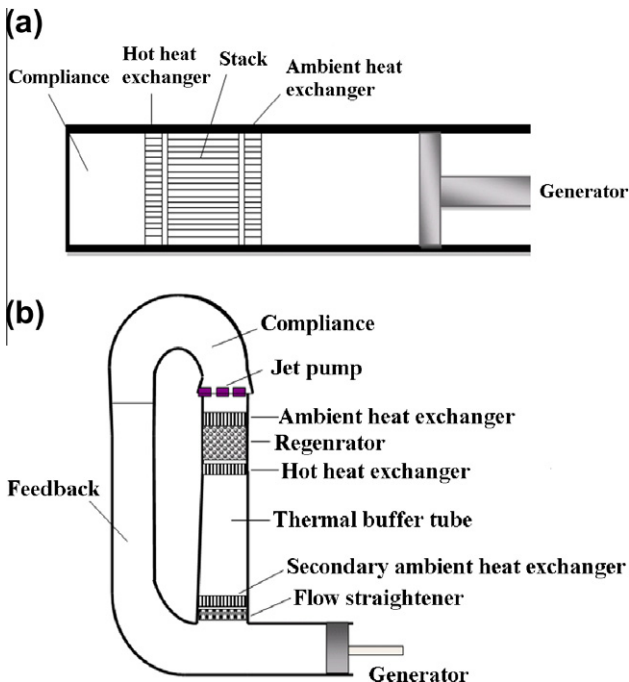


Fig. 1. General structure of a thermoacoustic engines: (a) standing-wave; (b) traveling-wave.

possible to decrease this flows by introducing additional bodies into the acoustic network (for example, jet pump dissymmetrical membranes, openings, conical tubes, etc.) but these elements cause, also, a pressure drop and friction dissipations of acoustic power.

Systematic studies on the causes and consequences of the energy losses, using the influence of the traveling–standing-wave ratio, become necessary to realize and design high efficiency thermoacoustic devices. Recently, it appeared in the specialized literature, some studies related to the influence of the traveling–standing-wave ratio or treating the energetic optimization of thermoacoustic systems. Kang et al. [21–23] have studied the thermoacoustic effect of traveling–standing-wave and they have done an optimization study of hydraulic radius of a thermoacoustic engine. Very often, the results of the author's works are based on the exclusive framework of a pure traveling-wave or standing-wave with a sound field presumably given within the resonator and not modified by the presence of the stack and the heat exchangers. Hu et al. [24] have studied the evaluation of thermal efficiency and energy conversion of thermoacoustic Stirling engines which is working with just a pure traveling-wave. Babaei and Siddiqui [25] have developed an optimization algorithm to design thermoacoustically-driven thermoacoustic refrigerators using a pure standing-wave thermoacoustic design. Yu and Jaworski [26] have studied the impact of acoustic impedance on the acoustic power produced in pure traveling-wave of thermoacoustic devices.

This paper tries to introduce more advances in the thermoacoustic research by indicating how the stack's hydraulic radius, the stack's position in the resonator, and the traveling–standing-wave ratio can modify the acoustic energy, and therefore the exergetic efficiency and most importantly the product between the acoustic energy and the exergetic efficiency of a thermoacoustic engine with a parallel-plate stack. For doing that, the thermoacoustic equations, developed by Rott and written by Swift [11], are rewritten in a dimensionless numbers in order to facilitate the calculus and the analyses. Next, synthetic optimization is represented to give a schematic description of the exergetic efficiency vs. the acoustic energy. Then, the Particle Swarm Optimization method (PSO) is introduced and used. So far in the literature, synthetic optimizations have been applied to the thermoacoustic research. Also, classical optimization methods such as the gradient-descend and the quasi-newton cannot be involved easily in the thermoacoustic optimization problems, because these ones are so complicated, irregular and non-linear. Consequently, classical and synthetic optimization methods need a huge amount of calculation time and space in the actual computers in order to optimize a function depending on more than two parameters, which can be frequently faced in the thermoacoustic research. In contrast, the PSO method is a powerful optimization method that reduces the calculation time and space and that could be easily used.

As a second novel contribution, the optimization of the product of the acoustic energy and the exergetic efficiency is studied. This leads to better results because it finds the best tradeoff between the two parameters of the product. This paper is ended up with a discussion of results and a significant conclusion that could be used as a guide to design any new thermoacoustic devices.

2. Acoustic power amplification and attenuation in thermoacoustic devices

The acoustic wave amplification was explained qualitatively a long time ago by Rayleigh [27]: “if heat be given to the air at the moment of greatest condensation [i.e., greatest density] or be taken from it at the moment of greatest rarefaction, the vibration is encouraged”. Actually, this phenomenon occurs only when an acoustic wave oscillates in a fluid in contact with a porous medium (stack or

regenerator), which has a sufficient temperature gradient. The time-averaged acoustic power $\dot{W}$ over a cross-sectional area “A” of the channel is calculated as:

$$\langle \dot{W} \rangle = \langle p_a \bar{u}_a \rangle = \frac{1}{2} \Re[\bar{p}_a \bar{u}_a] \quad (1)$$

where p_a is the oscillating pressure, $\bar{u}_a$ is the oscillating volume flow rate, which is the integral of the x components of the oscillating velocity u_a over the cross-sectional area A of the channel, $\Re[\]$ is the real part of the term between the brackets, $\langle \ \rangle$ denotes the time-average, the overdot denotes the time rate, the overbar denotes the spatial-average perpendicular to x , and the tilde denotes the complex conjugate.

The enthalpy flow is generally introduced to explain the physics and the dynamics of traditional Stirling machines, of pulse-tube refrigerators or of thermoacoustic devices. In fact, it is considering the first law of thermodynamics, which is called the conservation of energy principle. So, the acoustic approximation to the time-averaged total power $\langle \dot{H} \rangle$ over a cross-sectional area A can be expressed as:

$$\langle \dot{H} \rangle = \frac{1}{2} C_{pg} \bar{\rho}_g \int_A R[T_a \bar{u}_a] = \langle p_a \bar{u}_a \rangle + \bar{T}_g \langle \dot{S} \rangle = \langle \dot{W} \rangle + \langle \dot{Q} \rangle \quad (2)$$

where $\langle \dot{S} \rangle$ is the time-averaged entropy flux over a cross-sectional area A , C_{pg} is the fluid heat capacity at constant pressure, $\bar{\rho}_g$ is the fluid mean density, T_a is the oscillating temperature amplitude of the fluid, $\bar{T}_g$ is the fluid mean temperature and $\langle \dot{Q} \rangle$ is the time-averaged heat power over a cross-sectional area A . As shown by Eq. (2), the total power is equal to the sum of the acoustical (or mechanical) power and the heat power.

From Eq. (2), we can distinguish that when the fluid is oscillating within a porous media, characterized by its hydraulic radius r_h , which is much smaller than the thermal penetration depth δ_t (where $\delta_t = \sqrt{\frac{2k_g}{\omega \rho_g c_p}}$), which is the case of nearly ideal regenerators, the oscillating temperature in the fluid is almost null $T_a \simeq 0$, even if the oscillating pressure is not null. This means that the total power $\langle \dot{H} \rangle$ is also null and the acoustic power $\langle \dot{W} \rangle$ is always equal and opposite to the heat power $\langle \dot{Q} \rangle$. In a refrigerator or a heat pump, the consumption of the acoustic power $\langle \dot{W} \rangle$ induces the existence of a heat flow $\langle \dot{Q} \rangle$ that goes from the cold zone towards the hot zone. On the other hand, when the thermal penetration depth is much smaller than the hydraulic radius, and if there are no radial losses of heat in the fluid, so the system is in adiabatic situation and the total power remains constant.

As said previously, thermoacoustic devices can be classified as standing-wave stack-based devices or traveling-wave (acoustic-Stirling or pulse-tube) regenerator-based devices. Indeed, it should be remembered that a pure standing wave is a combination of traveling-wave and reflected-wave at same frequencies.

The regenerators and stacks must be sandwiched between two heat exchangers at different temperatures to work usefully. In fact, there is an intrinsic difference between the stack-based and regenerator-based devices. In the first one, the stack's hydraulic radius must be nearly equal to the thermal penetration depth ($r_h \simeq \delta_t$), in order to provide an imperfect thermal contact between the fluid and the stack's wall. This condition is needed to create the thermodynamic cycle, but the temporal phasing between the oscillating velocity and the oscillating pressure is close to 90° , which might carry no acoustic power and subsequently the efficiency is relatively low. In such devices, when the temperature gradient is higher than a critical mean temperature gradient, the stack produces an acoustic power without needing the presence of an initial acoustic power to create more acoustic power. In addition, the power producing phenomena in the stack is not altered by the presence

of acoustic power flowing through it in the either direction. Otherwise, when the temperature gradient is less than the critical mean temperature gradient, the acoustic power, flowing in any direction into the stack, is absorbed.

In the regenerator based devices, the hydraulic radius is much smaller than the thermal penetration depth ($r_h \ll \delta_t$) in order to satisfy the excellent thermal contact between the fluid and the regenerator's wall. This condition makes approximately an ideal thermodynamic cycle, so the temporal phasing between the oscillating velocity and the oscillating pressure is close to 0° and subsequently the efficiency of such devices is better than the stack-based devices. But the regenerator only amplifies or attenuates an existing acoustic power in it, so if there is no acoustic power incoming in the one end of the regenerator, there will not be an acoustic power outgoing in the other end. The acoustic power is amplified only if it flows from the cold to hot end of the regenerator, and the acoustic power amplification factor is equal to the ratio of the hot to cold temperature end of the regenerator. Contrariwise, the acoustic power is attenuated or absorbed in the regenerator, when it flows from the hot to cold temperature end of the regenerator. Because the high efficiency of regenerator-based devices, it is interesting to realize an acoustic network with a feedback acoustical loop with the possibility of modifying the traveling-standing-wave ratio.

3. Dimensionless of thermoacoustic equations

Since the length of the stack is too small in comparison to the wave length, the basic structure of the acoustic pressure field is assumed to be not fundamentally modified by the presence of the stack. Hence, the oscillating pressure propagating, in an ideal gas in the direction x of a channel with a cross-sectional area A , is supposed to be:

$$p_a(x)e^{j\omega t} = (1 - \tau)p_- + \tau p_+ = [p((1 - \tau)e^{jk_0x} + \tau e^{-jk_0x})]e^{j\omega t} \quad (3)$$

where τ is the traveling-standing-wave ratio, $0 \leq \tau \leq 1$, it is equal to 1 for a pure traveling-wave, 0.5 for a pure standing-wave, and 0 for a pure reflecting traveling-wave, p_a is the non-time dependent part of the oscillating pressure, p is the pressure amplitude, k_0 is the wave number, $k_0 = \frac{\omega}{c_0} = \frac{2\pi}{\lambda_0}$, and c_0 is the sound speed at the reference temperature T_0 , $c_0 = \sqrt{\gamma r T_0}$.

So, the pressure of a pure-standing wave, in an ideal acoustic channel, is written as:

$$p_a(x)e^{j\omega t} = \frac{1}{2}p_- + \frac{1}{2}p_+ = \left[\frac{1}{2}p(e^{jk_0x} + e^{-jk_0x}) \right] e^{j\omega t} \quad (4)$$

And the pressure of a pure and reflecting traveling-wave, in an ideal acoustic channel, are respectively expressed as:

$$\begin{cases} p_a(x)e^{j\omega t} = p_+ = [p(e^{-jk_0x})]e^{j\omega t} \\ p_a(x)e^{j\omega t} = p_- = [p(e^{jk_0x})]e^{j\omega t} \end{cases} \quad (5)$$

Next, the thermoacoustic equations derived by Rott and written by Swift [11] are rewritten in a dimensionless number.

3.1. Acoustic field

The pressure field supposed in Eq. (3) is rewritten by using definitions in Table 1, in a dimensionless number as:

$$p_a^* = DR((1 - \tau)e^{j2\pi x^*} + \tau e^{-j2\pi x^*}), \quad (6)$$

where DR is the drive ratio, $DR = \frac{p}{p_g}$.

The Rott's acoustic approximation to the x components of the momentum equation gives the relation between the oscillating

Table 1

Dimensionless number definitions of parameters.

Parameters	Parameters in dimensionless number
Position, x	$x^* = \frac{x}{\lambda_0}$
Length of the stack, L	$L^* = \frac{L}{\lambda_0}$
Pressure, p_a	$p_a^* = \frac{p_a}{p_g}$
Velocity, u_a	$u_a^* = \frac{u_a}{c_0}$
Temperature, T_h ; $\bar{T}_g$; $\bar{T}_a$	$T_h^* = \frac{T_h}{T_0}$; $\bar{T}_g^* = \frac{\bar{T}_g}{T_0}$; $\bar{T}_a^* = \frac{\bar{T}_a}{T_0}$
Density, ρ_g	$\rho_g^* = \frac{\rho_g}{\rho_0}$
Work flux, $\langle \dot{W}_x \rangle$	$\langle \dot{W}_x^* \rangle = \frac{\gamma \langle \dot{W}_x \rangle}{A p_g c_0}$
Work flux gradient, $\frac{d\langle \dot{W}_x \rangle}{dx}$	$\frac{d\langle \dot{W}_x^* \rangle}{dx^*} = \frac{\gamma \langle \dot{W}_x \rangle}{A p_g c_0}$
Heat flux, $\langle \dot{Q}_x \rangle$	$\langle \dot{Q}_x^* \rangle = \frac{\gamma \langle \dot{Q}_x \rangle}{A p_g c_0}$
Energy flux, $\langle \dot{H}_x \rangle$	$\langle \dot{H}_x^* \rangle = \frac{\gamma \langle \dot{H}_x \rangle}{A p_g c_0}$

velocity and the oscillating pressure gradient. This relation in a dimensionless number becomes:

$$\frac{\partial p_a^*}{\partial x^*} = \frac{\omega \lambda_0}{j c_0} \frac{\rho_g^*}{(1 - g_0(s_*))} u_a^*, \quad (7)$$

where $g_0(s_*)$ is the spatial averaged of a complex thermoacoustic function which depends on the specific channel geometric for two infinite parallel plates, $g_0(s_*) = \frac{\tanh(s_*)}{s_*}$, $s_* = \frac{r_h}{\delta_v} \sqrt{2j}$, where δ_v is the viscous penetration depth, $\delta_v = \sqrt{\frac{2\mu}{\omega \rho_g}}$, Pr is the Prandtl number,

$Pr = \frac{\delta_v^2}{\delta_t^2}$ and $\bar{\rho}_g$ is the mean density for an ideal gas, $\bar{\rho}_g = \frac{\bar{p}_g}{r \bar{T}_g} = \frac{\gamma \bar{p}_g}{c_{pg}(\gamma - 1) \bar{T}_g}$.

Transforming the Rott's acoustic approximation to the continuity equation into a dimensionless number, we deduce that:

$$\frac{du_a^*}{dx^*} = \frac{\omega \lambda_0}{j c_0} \left[1 + (\gamma - 1)g_0(s_* \sqrt{Pr}) \right] p_a^* + G_0 \frac{1}{\bar{T}_g^*} \frac{dT_g^*}{dx^*} u_a^* \quad (8)$$

where G_0 is a combined of the spatial averaged thermoacoustic functions,

$$G_0 = \frac{(g_0(s_*) - g_0(s_* \sqrt{Pr}))}{(Pr - 1)(1 - g_0(s_*))}.$$

3.2. Acoustic power

Because we are just interested by what is happening inside the stack, the studying zone is reduced to the stack only, given that the position of the studying zone varies in the resonator. So the position in the stack, x^* , varies between $[x_c^* - \frac{L^*}{2}; x_c^* + \frac{L^*}{2}]$, (see Fig. 2), where x_c^* is the center of the stack, and L^* is the length of the stack.

Under these circumstances, the acoustic power at position x^* is equal to:

$$\langle \dot{W}_x^* \rangle = \frac{1}{2} \Re [p_a^* u_a^*] = \frac{1}{2} \Re [\tilde{p}_a^* u_a^*] \quad (9)$$

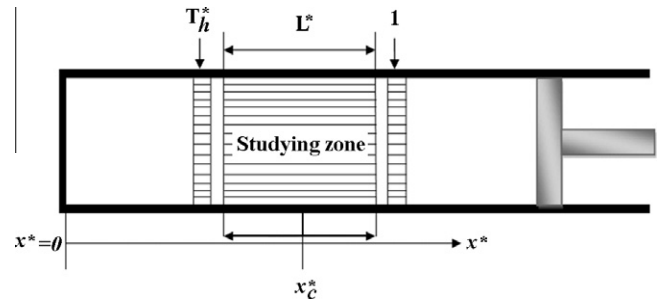


Fig. 2. The studying zone and the stack's position in the resonator.

The derivation of the time-averaged acoustic power $d\langle\dot{W}_x^*\rangle$ produced in a length dx^* of the channel is given as:

$$\frac{d\langle\dot{W}_x^*\rangle}{dx^*} = \frac{1}{\bar{T}_g} \frac{d\bar{T}_g^*}{dx^*} R[G_0] \langle\dot{W}_x^*\rangle + \frac{1}{2\bar{T}_g^*} \frac{d\bar{T}_g^*}{dx^*} I[-G_0] I[\bar{p}_a^* \bar{u}_a^*] - \frac{\lambda_0 \gamma \bar{p}_g}{2c_0 r_t} |\bar{p}_a^*|^2 - \frac{\lambda_0 c_0 r_v}{2\gamma \bar{p}_g} |u_a^*|^2 \quad (10)$$

where $\bar{T}_g^*$ is the gas mean temperature which is supposed to be linear throughout the stack, $\bar{T}_g^* = \frac{\bar{T}_g}{T_0} = T_h^* + \frac{(x^* - x_c^* + \frac{L}{2})}{L^*} (1 - T_h^*)$, and its axial thermal gradient $\frac{d\bar{T}_g^*}{dx^*} = \frac{(1 - T_h^*)}{L^*}$, where r_v and $\frac{1}{r_t}$ are the acoustic viscous resistance and the acoustic thermal resistance, respectively, $r_v = \omega \bar{\rho}_g \frac{\Im(-g_0(s_*))}{|1 - g_0(s_*)|^2}$ and $\frac{1}{r_t} = \frac{\gamma - 1}{\gamma} \frac{\omega}{\bar{p}_g} \Im(-g_0(s_* \sqrt{Pr}))$.

3.3. Heat power

The heat flux, $\langle\dot{Q}_x^*\rangle$, which is a combination of two terms, an internal heat flux and a thermal-conduction, is equal to:

$$\langle\dot{Q}_x^*\rangle = \langle\dot{Q}_{internal}^*\rangle + \langle\dot{Q}_{cond}^*\rangle \quad (11)$$

$$\langle\dot{Q}_x^*\rangle = \left(\frac{c_0}{2\omega\lambda_0} \frac{1}{\rho_g^*} \Im \left[\frac{d\bar{p}_a^*}{dx^*} \bar{p}_a^* (\tilde{g}_0(s_*) - \psi) \right] + \frac{\gamma \bar{p}_g T_0}{c_0 \lambda_0^3} k_{eq}^* \frac{1}{\rho_g^*} \frac{d\bar{T}_g^*}{dx^*} \right) - \frac{\gamma T_0}{\bar{p}_g c_0 \lambda_0} \left[(1 - \varepsilon) k_s \frac{d\bar{T}_g^*}{dx^*} + \varepsilon k_g \frac{d\bar{T}_g^*}{dx^*} \right], \quad (12)$$

3.4. Total power

For an ideal gas and for a material with high heat capacity, the total power is the sum of the acoustic power and heat power according to the first law of thermodynamics, and is expressed as:

$$\langle\dot{H}_x^*\rangle = \langle\dot{W}_x^*\rangle + \langle\dot{Q}_x^*\rangle \quad (13)$$

$$\langle\dot{H}_x^*\rangle = \frac{c_0}{2\omega\lambda_0 \rho_g^*} \Im \left[\frac{d\bar{p}_a^*}{dx^*} \bar{p}_a^* (1 - \psi) \right] + \frac{\gamma \bar{p}_g T_0}{c_0 \lambda_0^3} k_{eq}^* \frac{1}{\rho_g^*} \frac{d\bar{T}_g^*}{dx^*} - \frac{\gamma T_0}{\bar{p}_g c_0 \lambda_0} \left[(1 - \varepsilon) k_s \frac{d\bar{T}_g^*}{dx^*} + \varepsilon k_g \frac{d\bar{T}_g^*}{dx^*} \right], \quad (14)$$

where

$$k_{eq}^* = \frac{c_{pg} \Im[\psi]}{2\omega^3 \rho_0 (1 - Pr)} \left| \frac{d\bar{p}_a^*}{dx^*} \right|^2, \rho_g^* = \frac{\bar{p}_g}{\rho_0} = \frac{T_0}{\bar{T}_g^*} \quad \text{and} \quad \rho_0 = \frac{\bar{p}_g}{r T_0}, \quad \text{and} \quad \psi = \tilde{g}_0(s_*) + \frac{(g_0(s_* \sqrt{Pr}) - \tilde{g}_0(s_*))}{(1 + Pr)}.$$

4. Exergetic efficiency of a thermoacoustic engine

Approximately, the acoustic power produced or consumed by a stack or a regenerator can be calculated as the sum, throughout the stack, of the derivation of the time-averaged acoustic power $d\langle\dot{W}_x^*\rangle$ produced in a length dx^* , so:

$$\Delta\langle\dot{W}^*\rangle = \frac{L^*}{N} \sum_{i=1}^N \frac{d\langle\dot{W}_x^*\rangle}{dx^*} \Big|_{x_{ci}^*} = \frac{L^*}{N} \sum_{i=1}^N \left(\frac{1}{\bar{T}_g^*} \frac{d\bar{T}_g^*}{dx^*} R[G_0] \langle\dot{W}_x^*\rangle + \frac{1}{2\bar{T}_g^*} \frac{d\bar{T}_g^*}{dx^*} I[-G_0] I[\bar{p}_a^* \bar{u}_a^*] - \frac{\lambda_0 \gamma \bar{p}_g}{2c_0 r_t} |\bar{p}_a^*|^2 - \frac{\lambda_0 c_0 r_v}{2\gamma \bar{p}_g} |u_a^*|^2 \right) \Big|_{x_{ci}^*} \quad (15)$$

where N is the discretization number of the stack, x_{ci}^* is the center position of the i th interval step of the discretized stack, and $\frac{d\langle\dot{W}_x^*\rangle}{dx^*}$ is expressed in Eq. (10).

Supposing that a system receives a quantity $\langle\dot{Q}_h^*\rangle$ of heat from a heat source which has an imposed hot temperature T_h^* , and the system produces a quantity $-\Delta\langle\dot{W}^*\rangle$ of acoustic power, then the energy efficiency of the system is:

$$\eta_{en} = \frac{-\Delta\langle\dot{W}^*\rangle}{\langle\dot{Q}_h^*\rangle} \quad (16)$$

And therefore the exergetic efficiency is defined as:

$$\eta_{ex} = \frac{\eta_{en}}{\eta_{Carnot}} = \frac{-T_h^* \Delta\langle\dot{W}^*\rangle}{(T_h^* - 1) \langle\dot{Q}_h^*\rangle} \quad (17)$$

where η_{Carnot} is the Carnot efficiency, $\langle\dot{Q}_h^*\rangle$ is the heat added to the system and can be calculated by using Eq. (12):

$$\langle\dot{Q}_h^*\rangle = \left(\frac{c_0}{2\omega\lambda_0} \frac{1}{\rho_g^*} \Im \left[\frac{d\bar{p}_a^*}{dx^*} \bar{p}_a^* (\tilde{g}_0(s_*) - \psi) \right] + \frac{\gamma \bar{p}_g T_0}{c_0 \lambda_0^3} k_{eq}^* \frac{1}{\rho_g^*} \frac{d\bar{T}_g^*}{dx^*} \right) \Big|_{x_h^*} \quad (18)$$

where x_h^* is the position of the hot heat exchanger in the resonator.

Replacing Eqs. (15) and (18) into Eq. (17), the exergetic efficiency becomes:

$$\eta_{ex} = \frac{-T_h^*}{(T_h^* - 1)} \frac{\sum_{i=1}^N \left(\frac{1}{\bar{T}_g^*} \frac{d\bar{T}_g^*}{dx^*} R[G_0] \langle\dot{W}_x^*\rangle + \frac{1}{2\bar{T}_g^*} \frac{d\bar{T}_g^*}{dx^*} I[-G_0] I[\bar{p}_a^* \bar{u}_a^*] - \frac{\lambda_0 \gamma \bar{p}_g}{2c_0 r_t} |\bar{p}_a^*|^2 - \frac{\lambda_0 c_0 r_v}{2\gamma \bar{p}_g} |u_a^*|^2 \right) \Big|_{x_{ci}^*}}{\left(\frac{c_0}{2\omega\lambda_0} \frac{1}{\rho_g^*} \Im \left[\frac{d\bar{p}_a^*}{dx^*} \bar{p}_a^* (\tilde{g}_0(s_*) - \psi) \right] + \frac{\gamma \bar{p}_g T_0}{c_0 \lambda_0^3} k_{eq}^* \frac{1}{\rho_g^*} \frac{d\bar{T}_g^*}{dx^*} \right) \Big|_{x_h^*}} \quad (19)$$

5. Synthetic optimization

In this part, a synthetic optimization is represented and discussed. The idea that lies behind synthetic optimization is to determine how to modify the exergetic efficiency as a function of the acoustic energy according to different values of r_h , τ and x_c^* while all the other parameters remains fixed and according to two different working gases Helium and Air. The mean pressure is fixed at 25 bar, the dimensionless of the stack's length L^* is equal to 0.0025 and the hot and cold temperatures are respectively set to be 893 K and 293 K. In order to using the same size stack and subsequently the same boundary layer condition for both working gases, Helium and Air, the frequency for Helium is chosen to be 50 Hz which corresponds to 6.5 Hz for Air. The drive ratio, DR is supposed to be 5%. In fact, when the drive ratio is fixed, this leads to say that the size of heat exchangers is modified to match the desire drive ratio after finding the optimized values.

The variable parameters are:

The center of the stack x_c^* , which varies between $\frac{L^*}{2}$ and $\frac{L^*}{2} + 0.5$. By using dimensionless number, the wavelength is excluded from use in the calculation, which simplifies the comparison between both gases used in this paper.

The hydraulic radius r_h has three different values 0.15 mm, 0.2 mm, and 0.31 mm, where the thermal penetration depth is in order of $\delta_t \approx 0.3$ mm.

The traveling-standing wave ratio τ has three different values 0.6, 0.75 and 0.95. τ depends on the boundary conditions of the resonator. So, after finding the optimized value of τ , the boundary conditions will be chosen to match the desired τ .

After highlighting the data, Figs. 3 and 4 are resulting.

As shown in Figs. 3 and 4, the acoustic power is strongly depending on the stack's position, the stack's hydraulic radius and the traveling-standing-wave ratio. As results show, at fixed τ

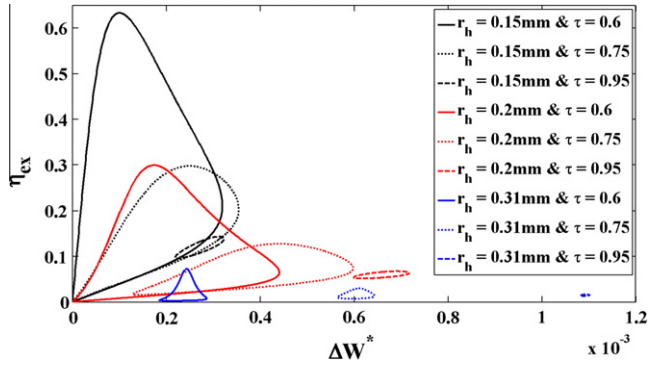


Fig. 3. Synthetic optimization of η_{ex} vs. $\Delta(W^*)$ for Helium as working gas at frequency 50 Hz.

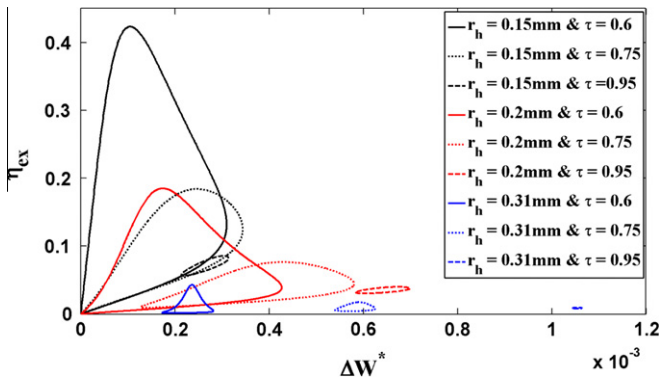


Fig. 4. Synthetic optimization of η_{ex} vs. $\Delta(W^*)$ for Air as working gas at frequency 6.5 Hz.

and r_h , the acoustic power varies in an interval range having a maximum and a minimum, depending on the stack's position in the resonator. This explains the variation of the oscillating pressure $p_a^*(x^*)$ and oscillating velocity $u_a^*(x^*)$ inside the resonator. The length of the interval range of the acoustic power is changed with the variation of τ and r_h . For the highest acoustic power, as Figs. 3 and 4 show, the traveling-standing-wave ratio must be near a pure traveling-wave when τ is equal to 1, and the stack's hydraulic radius, r_h , must be in the order of the thermal penetration depth $r_h \approx \delta_t$ which is approximately equal to 0.3 mm. The exergetic efficiency that corresponds to the highest acoustic power, which is approximately equal to 0.0011, is quite low and it is in the order of $\eta_{ex} \approx 1\%$.

The highest exergetic efficiency is obtained at a traveling-standing-wave ratio near to a pure standing-wave, that is for $\tau = 0.5$, and for a stack's hydraulic radius less than the thermal penetration depth, i.e. for $r_h \ll \delta_t$. In this case, the exergetic efficiency has also an interval range with maximum and minimum values that depend on the stack's position in the resonator. But the length of this interval is tending to be zero, or in other words, the exergetic efficiency becomes independent of the stack's position in the resonator when τ is a pure traveling-wave, i.e. when $\tau = 1$, and when r_h is in the order of thermal penetration depth, i.e. $r_h \approx \delta_t$. At fixed τ and for r_h varied increasingly, or at fixed r_h and for τ varied increasingly, the highest exergetic efficiency is decreased. The acoustic power corresponding to the highest exergetic efficiency, which is approximately equal to 75% for Helium and 50% for Air, is quite low and it is in the order of $\Delta W^* \approx 0.0001$.

As we said previously, the frequency for helium and air as working gases must be equal to 50 Hz and 6.5 Hz respectively in order to obtain the same stack size and subsequently the same boundary

layer conditions. In other word, a wholesale machine with Air as working gas is 7.7 times longer than a machine working with Helium. And we also note that the ratio of the acoustic power with Helium to the acoustic power with Air is equal to:

$$\left(\frac{\bar{p}_g C_0}{\gamma} \right)_{\text{helium}} / \left(\frac{\bar{p}_g C_0}{\gamma} \right)_{\text{air}} = 1.5116 \times 10^9 / 6.1270 \times 10^8 \simeq 2.5 \quad (27)$$

In other word, the acoustic power produced by the same stack using helium is 2.5 better than using the air.

6. Optimization using the Particle Swarm method (PSO)

The Particle Swarm Optimization (PSO) is an iterative method that tries to optimize a function or a problem. Every problem has multiple candidate solutions, called particles, which are characterized by their positions and velocities over a mathematical formula, in a known search-space of dimension N . Every particle saves its own best position overall iterations and saves also the best position found so far in the search-space by which all the swarm (the group of particles) is expected to move toward it.

The Particle Swarm Optimization (PSO) method was invented by Kennedy and Eberhart [28]. The PSO is then used when classical optimization methods such as gradient descend and quasi-newton cannot be applied because of the complexity, irregularity or non-linearity of the problem. In addition, the PSO needs just few or no assumptions to optimize any problem and it can be employed in a large search-space dimension problem. Because the PSO method is a stochastic method, it may not converge to the desired optimal solution from the first simulation, and hence, it is better to repeat the optimization procedure more than twice under the same conditions to ensure that the function is converging to the right value.

6.1. Algorithm

We define the cost function to be optimized (minimized or maximized) as:

$$f : \begin{cases} \mathbb{R}^N \rightarrow \mathbb{R} \\ \vec{x} \rightarrow f(\vec{x}) \end{cases}$$

where $\vec{x}$ is the position vector of dimension N . Here, N represents the dimension of the search-space.

Let P be the number of candidate solutions (particles) in the swarm, where each particle has a position vector of dimension N . So we define $\vec{x}_i; i = 1, \dots, P$ as the position vector of a particle i and $\vec{v}_i$ as its velocity.

Let k be the number of iterations, so the algorithm for maximizing a function can be summarized as:

- I. for each iteration $j = 1, \dots, k$,
 1. for each particle $i = 1, \dots, P$,
 - i. Evaluate the fitness value of the cost function, $f_j^i(\vec{x}_i)$ at position $\vec{x}_i$.
 - ii. If $f_j^i(\vec{x}_i) > f_{\text{best}}^i(\vec{x}_{\text{best}}^i)$, then $f_{\text{best}}^i(\vec{x}_{\text{best}}^i) = f_j^i(\vec{x}_i)$ and $\vec{x}_{\text{best}}^i = \vec{x}_i$, where $f_{\text{best}}^i(\vec{x}_{\text{best}}^i)$ is the best value retained by a particle i at position $\vec{x}_{\text{best}}^i$.
 - iii. If $f_k^g(\vec{x}_j) > f_{\text{best}}^g(\vec{x}_{\text{best}}^g)$, then $f_{\text{best}}^g(\vec{x}_{\text{best}}^g) = f_k^g(\vec{x}_j)$ and $\vec{x}_{\text{best}}^g = \vec{x}_j$, where $f_{\text{best}}^g(\vec{x}_{\text{best}}^g)$ is the global best value retained at position $\vec{x}_{\text{best}}^g$.
 - iv. Update particle position $\vec{x}_{j+1}^i$ and velocity $\vec{v}_{j+1}^i$.
 - v. If stopping conditions are satisfied go to step II.

II. Report results and terminate.

6.2. Defining cost functions to be optimized

We are interesting by maximize three functions:

- the first one is the exergetic efficiency of the thermoacoustic engine η_{ex} ,
- the second one is the acoustic power produced by the stack of a thermoacoustic engine, $\Delta\langle W^* \rangle$,
- And the last function is the product between the acoustic power produced by the stack and the exergetic efficiency, $\eta_{ex} \times \Delta\langle W^* \rangle$. This function is useful specially to find a tradeoff between the best of η_{ex} and $\Delta\langle W^* \rangle$. Because the last section has shown that when η_{ex} is in its maximum, $\Delta\langle W^* \rangle$ is in its minimum, and vice versa, so, it is better to design a thermoacoustic device based on this function.

6.3. Defining search-space and parameters of the problem

For the first time, the PSO is involved in the thermoacoustic research, so in this paper we concern ourselves by studying a 3D search-space dimension problem.

The cost functions studied in this paper depend on three parameters:

- The hydraulic radius of a parallel-plate stack, r_h ,
- The center of the stack in the resonator, x_c^* , and
- The traveling-standing-wave ratio, τ .

These three parameters are studied to see their influence on the maximum cost functions mentioned above, η_{ex} , $\Delta\langle W^* \rangle$ and $\eta_{ex} \times \Delta\langle W^* \rangle$.

The search-space of particles is summarized in Table 2. For x_c^* , the interval range corresponds to the stack's position varying from zero to the half of a wavelength. The hydraulic radius, r_h , is lying between 0.15 mm and 0.31 mm where the thermal penetration depth, δ_t , is in order of 0.3 mm, the second value is in order of δ_t , and the last value is greater than δ_t . The traveling-standing-wave ratio, τ , will be varied between 0.5, which corresponds to a pure standing-wave ratio, and 1, which corresponds to a pure traveling-wave ratio. In this paper, we concern ourselves in what is happening inside the stack of a thermoacoustic engine. Then, the boundary conditions and the size of the heat exchangers are chosen to match the optimized values obtained.

Table 3 summarizes data of fixed parameters concerning cost functions for each of the two working gases studied, Helium and Air. The mean pressure of the system is supposed to be 25 bar, the hot and cold temperature are equal to 893 K and 293 K respectively, while the dimensionless length of the stack is set to be 0.0025. The drive ratio is fixed also at 5%. We suppose that the drive ratio is fixed in order to say in other words that the size of the hot and cold heat exchanger can be changed to maintain the desire value of drive ratio. Thermophysical properties of helium and air used in this paper are also found in Table 3. Concerning the frequency, the thermoacoustic engine is working at 50 Hz when Helium is used and at 6.5 Hz when Air is used. The difference in the frequency between Helium and Air aims at maintaining the same stack size and subsequently the same boundary layer conditions.

Table 2
Search-space for optimizing.

Parameter	Symbol (unit)	Helium	Air
Dimensionless number of center of the stack	x_c^*	$[\frac{L}{2}; \frac{L}{2} + 0.5]$	$[\frac{L}{2}; \frac{L}{2} + 0.5]$
Hydraulic radius	r_h (mm)	[0.15; 0.31]	[0.15; 0.31]
Traveling-standing-wave ratio	τ	[0.5; 1]	[0.5; 1]

Table 3

Parameters concerning functions to be optimized.

Parameters	Symbol (unit)	Helium	Air
Mean pressure	$\bar{p}_g$ (bar)	25	25
Hot and cold temperature	T_h, T_0 (K)	893, 293	893, 293
Dimensionless number of length of stack	L^*	0.0025	0.0025
Frequency	fr (Hz)	50	6.5
Drive ratio	DR	5%	5%
Dynamic viscosity	μ (kg m s ⁻¹)	0.000018	0.0000175
Gas constant	R (J Kg ⁻¹ K ⁻¹)	2080	287
Thermal conductivity	k_g (W m ⁻¹ K ⁻¹)	0.1392	0.02415
Constant pressure specific heat	C_p (J Kg ⁻¹ K ⁻¹)	5183	1005
Specific heat ratio	γ	1.667	1.4

Table 4

PSO parameters.

PSO parameters	
Maximum number of iterations (K)	$K = 2,000,000$
Number of particles (P)	$P = 24$
Stopping conditions	If, for 250 iterations, the variation of global best, $f_{best}^g(x_{best}^g)$ is less than $1-e25$, then stop the simulation

Regarding the particle swarm optimization method, we will refer to the particle swarm optimization toolbox developed by Birge [29]. Table 4 summarizes initialization parameters of PSO toolbox used for our optimizations case.

6.4. Optimization results and discussion

The particle swarm optimization method is involved for the first time in the thermoacoustic research in this paper. Hence, to be confident that this method works well, two verification methods are done:

- As the PSO method is a stochastic method, 10 trials were done, under the same conditions, for each optimization function in order to ensure that the results were converging to the right solution and to avoid the stagnation problem of such optimization method.
- In addition, optimization of the exergetic efficiency and acoustic power are studied in order to compare them with the results existing in the literature and therefore to guarantee that the powerful PSO method works well.

Then, as a novel contribution, the PSO method is used to optimize a third function for the first time in the literature, i.e. the product $\eta_{ex} \times \Delta\langle W^* \rangle$, the product between the exergetic efficiency and the acoustic power.

6.4.1. Exergetic efficiency optimization $\eta_{ex}(r_h, x_c^*, \tau)$

Figs. 5 and 6 show the evolution of the exergetic efficiency of a thermoacoustic engine working with Air and Helium, respectively. As results shown in Table 5, the best values of the exergetic efficiency of a thermoacoustic engine are 50.14% for Air and 74.94% for Helium. It is well known that the thermoacoustic devices working with Helium are better than Air, because the thermophysical characteristics of Helium, which is a noble gas. The best values of the exergetic efficiency correspond to:

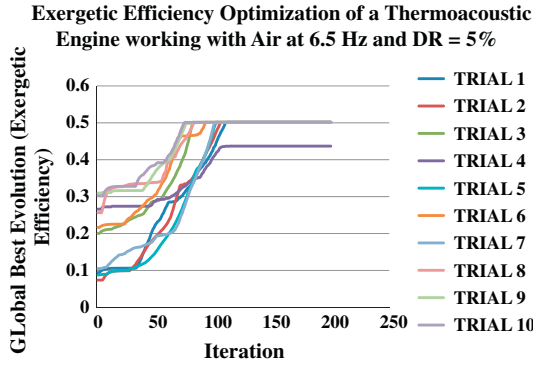


Fig. 5. Evolution of the exergetic efficiency of a thermoacoustic engine working with Air. Here, $\eta_{ex}(x_c^* = 0.011; r_h = 0.150 \text{ mm}; \tau = 0.5) = 50.14\%$.

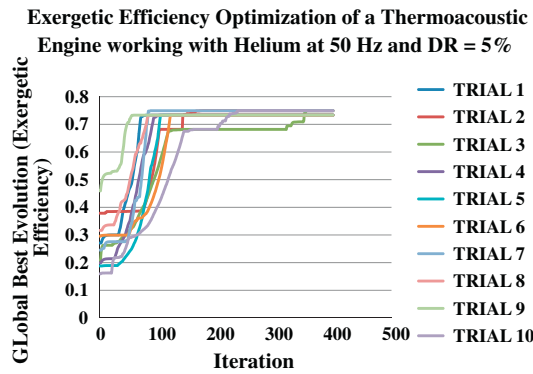


Fig. 6. Evolution of the exergetic efficiency of a thermoacoustic engine working with Helium. Here, $\eta_{ex}(x_c^* = 0.464; r_h = 0.150 \text{ mm}; \tau = 0.5) = 74.94\%$.

- Pure standing-wave, $\tau = 0.5$, which means, for example, a resonator tube with two closed end boundary conditions.
- Hydraulic radius less than the thermal penetration depth, δ_h . In our case, $r_h = 0.5 \text{ mm}$, where $\delta \approx 0.3 \text{ mm}$.
- Relative position of the stack in the resonator, x_c^* , which is equal to 0.011 when Air is the working gas and 0.46 for Helium.

The dimensionless acoustic power values corresponding to the best value of the exergetic efficiency are relatively low, 82.4×10^{-6} for Air and 69.8×10^{-6} for Helium. If we are taking into account Eq. (27), we find that the ratio of the acoustic power for Helium to Air corresponding to the best value of exergetic efficiency is equal to

$$\frac{\Delta \langle W_{Helium} \rangle}{\Delta \langle W_{Air} \rangle} \approx 2.5 \frac{\Delta \langle \dot{W}_{xc, Helium}^* \rangle}{\Delta \langle \dot{W}_{xc, Air}^* \rangle} \rightarrow \Delta \langle W_{Helium} \rangle \approx 2.1 \times \Delta \langle W_{Air} \rangle$$

The obtained results, for the exergetic efficiency optimization, are logic and in accordance with those that can be found in the literature. Kang et al. [23] shows that the best value of exergetic efficiency decreases as the traveling-standing-wave ratio increases as shown in this paper. Kang defines the pressure field in a way that the traveling-standing-wave ratio, τ , is equal to 0 for a pure standing-wave and 1 for a pure traveling-wave. Kang's definition of τ is different from that discussed in this paper, but the consequences on the exergetic efficiency are the same regardless of the representation of the pressure field.

6.4.2. Acoustic power optimization $\Delta \langle W_{xc}^* \rangle(r_h, x_c^*, \tau)$

For the highest acoustic power, the dimensionless number of acoustic power's best value obtained in this paper are equal to

Table 5

Exergetic efficiency optimization results.

	Parameters			Exergetic efficiency	Acoustic power
	x_c^*	r_h (mm)	τ	$\eta_{ex} (\%)$	$\Delta \langle \dot{W}_{xc}^* \rangle$
For Air	0.011	0.0150	0.5	50.14	0.0000824
For Helium	0.464	0.150	0.5	74.94	0.0000698

0.001188 for Air and 0.001228 for Helium, which, by using Eq.

(27), are equivalent to $\frac{\Delta \langle W_{Helium} \rangle}{\Delta \langle W_{Air} \rangle} \approx 2.5 \frac{\Delta \langle \dot{W}_{xc, Helium}^* \rangle}{\Delta \langle \dot{W}_{xc, Air}^* \rangle} \rightarrow \Delta \langle W_{Helium} \rangle \approx 2.6 \times \Delta \langle W_{Air} \rangle$. These results are shown in Figs. 7 and 8 and are summarized in Table 6.

The best values of the acoustic power correspond to:

- Pure traveling-wave, $\tau = 1$, which means, for example, a circled resonator tube.
- Hydraulic radius is in order of the thermal penetration depth, δ_t , in our case, $r_h \approx \delta_t \approx 0.3 \text{ mm}$.

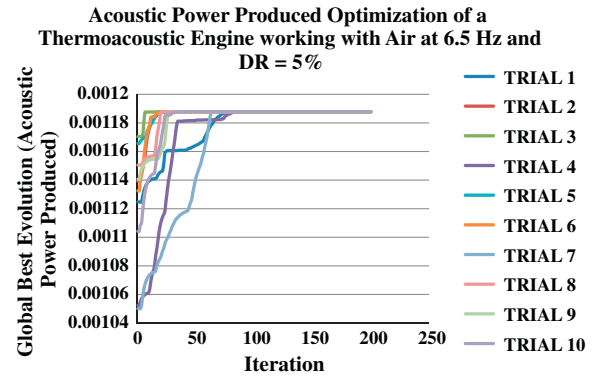


Fig. 7. Evolution of the acoustic power of a thermoacoustic engine working with Air. Here, $\Delta \langle W_{xc}^* \rangle(x_{ce}^* = 0.2; r_h = 0.291 \text{ mm}; \tau = 1) = 0.001188$.

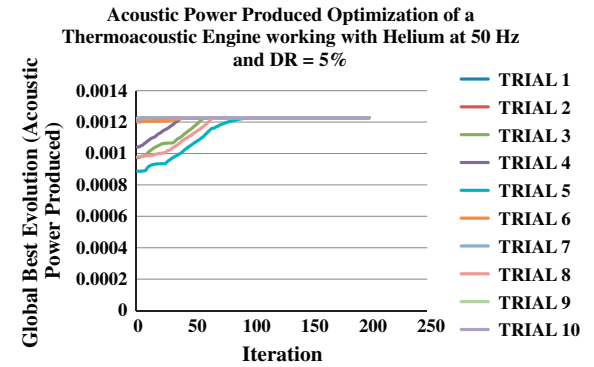


Fig. 8. Evolution of the exergetic efficiency of a thermoacoustic engine working with Helium. Here, $\Delta \langle W_{xc}^* \rangle(x_{ce}^* = 0.15; r_h = 0.293 \text{ mm}; \tau = 1) = 0.001228$.

Table 6

Acoustic power optimization results.

	Parameters			Exergetic efficiency	Acoustic power
	x_c^*	r_h (mm)	τ	$\eta_{ex} (\%)$	$\Delta \langle \dot{W}_{xc}^* \rangle$
For Air	0.2	0.291	1	1.07	0.001188
For Helium	0.15	0.293	1	1.8	0.001228

Table 7

Acoustic power times exergetic efficiency optimization results.

	Parameters			Exergetic efficiency $\eta_{ex} (\%)$	Acoustic power $\Delta \langle W_{ac}^* \rangle$	Exergetic efficiency times Acoustic power $\eta_{ex} \times \Delta \langle W_{ac}^* \rangle$
	x_c^*	r_h (mm)	τ			
For Air	0.0516	0.150	0.61	29.1	0.0002166	0.000063
For Helium	0.0587	0.150	0.63	39.61	0.0002494	0.0000988

- Position of the stack in the resonator, x_c^* , which is equal to 0.2 when Air is working gas and 0.15 for Helium.

As Table 6 shows, the exergetic efficiencies, for both Air and Helium, that correspond to the best value of acoustic power are quite low. These results are in conformity with the work of Kang et al. [23]. Kang found that the best value of the normalized acoustic power gain is when the traveling–standing-wave ratio $\tau = 0.68$. But this value is not matching the best value of the acoustic power itself. To calculate the acoustic power gain in the Kang's paper, we must multiply the normalized acoustic power gain by $\frac{p^2}{4T\rho_g c^2 - 2\rho} (1 + \tau^2)$. If this transfer condition is taken into account, we will find that the best value of the acoustic power gain is when the wave is a pure traveling, $\tau = 1$.

6.4.3. Exergetic efficiency times acoustic power optimization

In the previous simulations, we saw that the best value of exergetic efficiency has an acoustic power relatively low, while the best value of the acoustic power is when the exergetic efficiency is very low. So it might not be useful to design a thermoacoustic device based only on the best value of the exergetic efficiency or on the acoustic power. Hence, the idea is to optimize the product between the exergetic efficiency and acoustic power which seems to be a tradeoff solution between the best values of the acoustic power and exergetic efficiency. Optimization results of this function are shown in Table 7. The best value of this product is when the traveling–standing-wave ratio is approximately 0.62. So, it is necessary to adjust the boundary condition of the thermoacoustic engine to match this traveling–standing-wave ratio. Figs. 9 and 10 show the evolution of the best value of the product of acoustic power and exergetic efficiency.

Backhaus and Swift [14] have presented an empirical study in which they improved the exergetic efficiency of a thermoacoustic engine by making a traveling–standing-wave ratio between a pure traveling and standing wave and by making the stack's hydraulic radius shorter than the thermal penetration depth. But they did not give any explanation about the value of the traveling–standing-wave ratio neither about the optimized function. The mathematical optimization study presented in this paper has led to results (see Table 6) that are in conformity with those of [14]. Although this proves that Backhaus and Swift were in the right path, our mathematical study is general and can be applied to a wide range of empirical systems. Furthermore, this shows the importance of the contributions of this paper: the use of PSO and the optimization of the product function described before.

7. Conclusions

In this work, an optimization method, the particle swarm optimization method (PSO), for the thermoacoustic acoustic power and exergetic efficiency is represented and analyzed. It takes into account the effect of the stack's hydraulic radius, the stack's position in the resonator and the traveling–standing-wave ratio.

As a result, the optimal solution of the acoustic power is obtained when the wave is a pure traveling and when the hydraulic radius is in

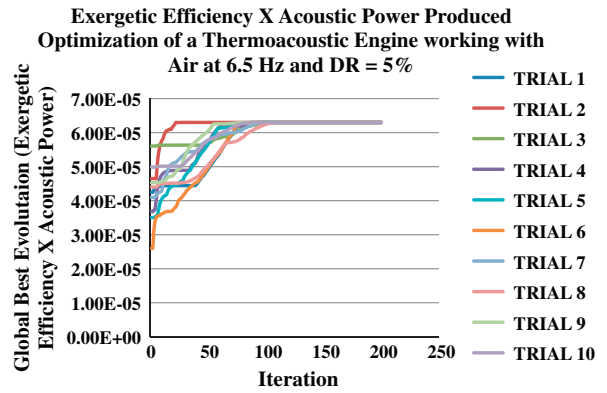


Fig. 9. The best value of the exergetic efficiency times acoustic power of a thermoacoustic engine working with Air. Here, $\Delta \langle W_{ac}^* \rangle \times \eta_{ex}(x_{ce}^* = 0.0516; r_h = 0.150 \text{ mm}; \tau = 0.61) = 0.000063$.

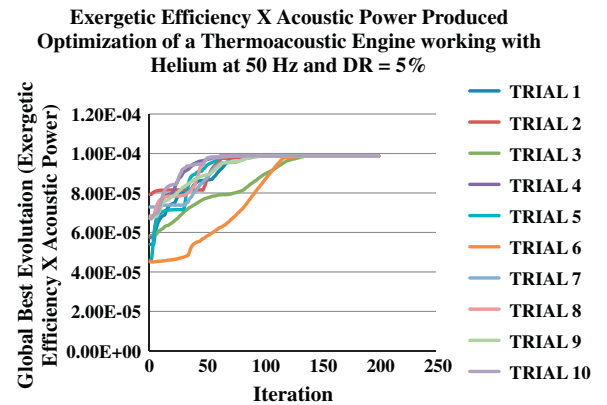


Fig. 10. the best value of the exergetic efficiency times acoustic power of a thermoacoustic engine working with Helium. Here, $\Delta \langle W_{ac}^* \rangle \times \eta_{ex}(x_{ce}^* = 0.0587; r_h = 0.150 \text{ mm}; \tau = 0.63)$.

the order of the thermal penetration depth. This leads to a circled resonator tube with a stack having a hydraulic radius in the order of the thermal penetration depth. When the wave is a pure traveling wave, this means that the oscillating pressure and velocity are in phase which explains why the acoustic power is maximal when the wave is a pure traveling-wave. But, in this case, the exergetic efficiency is quite low because the thermal and viscous dissipations are high.

For the highest exergetic efficiency, the thermoacoustic engine must work in a pure standing-wave, while the hydraulic radius should be less than the penetration depth which means that the thermal dissipation is low and this explains the highest efficiency. The resonator, in this case, has two closed end in order to obtain the standing-wave motion. Since the thermoacoustic engine, in this case, works at standing-wave, this leads to say that the oscillating pressure and velocity have 90° phase difference which gives explanation for the lowest value of the acoustic power.

Because it is not much useful to build a thermoacoustic engine based only on higher exergetic efficiency with lower acoustic power

or on higher acoustic power with lower exergetic efficiency, the product between acoustic power and exergetic efficiency is studied also. This function is in fact more realistic, because it leads to obtain a good exergetic efficiency and acoustic power. The corresponding traveling-standing-wave ratio is in the order of 0.6 while the hydraulic radius is shorter than the thermal penetration depth.

After obtaining interesting results by using the PSO method, our future work will focus on studying more complicated thermoacoustic devices by trying to take into account a large search-space dimension problem.

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References

- [1] Garrett SL. Resource letter: Ta-I: thermoacoustic engines and refrigerators. *Am J Phys* 2004;72:11–7.
- [2] Swift GW. Thermoacoustic engines and refrigerators. *Phys Today* 1995;48:22–8.
- [3] Rott N. Damped and thermally driven acoustic oscillations in wide and narrow tubes. *J Appl Math Phys (ZAMP)* 1969;20:230–43.
- [4] Rott N. Thermally driven acoustic oscillations, Part III: Second-order heat flux. *J Appl Math Phys (ZAMP)* 1975;26:43–9.
- [5] Rott N. Thermally driven acoustic oscillations, Part II: Stability limit for helium. *J Appl Math Phys (ZAMP)* 1973;24:54–72.
- [6] Rott N. Thermally driven acoustic oscillations, Part IV: Tubes with variable cross-section. *J Appl Math Phys (ZAMP)* 1976;27:197–224.
- [7] Rott N. Thermally driven acoustic oscillations, Part V: Gas-liquid oscillations. *J Appl Math Phys (ZAMP)* 1976;27:325–34.
- [8] Rott N. Thermally driven acoustic oscillations, Part VI: Excitation and power. *J Appl Math Phys (ZAMP)* 1983;34:609–26.
- [9] Rott N. Thermoacoustics. *Adv Appl Mech* 1980;20:135–75.
- [10] Swift GW. Thermoacoustic engines. *J Acoust Soc Am* 1988;84:1145–80.
- [11] Swift G. Thermoacoustics: a unifying perspective for some engines and refrigerators; 2001.
- [12] Wheatley J, Hoffer T, Swift GW, Migliori A. Understanding some simple phenomena in thermoacoustics with applications to acoustical heat engines. *Am J Phys* 1985;53:147–62.
- [13] Ceperley PH. A pistonless Stirling engine—the traveling wave heat engine. *J Acoust Soc Am* 1979;66:1508–13.
- [14] Backhaus S, Swift GW. A thermoacoustic-Stirling heat engine: detailed study. *J Acoust Soc Am* 2000;107:3148–66.
- [15] Backhaus S, Swift GW. A thermoacoustic Stirling heat engine. *Nature* 1999;399:335–8.
- [16] Backhaus S, Swift GW. Acoustic streaming instability in thermoacoustic devices utilizing jet pumps. *J Acoust Soc Am* 2003;113:1317–24.
- [17] Swift GW, Gardner DL, Backhaus S. Acoustic recovery of lost power in pulse tube refrigerators. *J Acoust Soc Am* 1999;105:711–24.
- [18] Gardner DL, Swift GW. A cascade thermoacoustic engine. *J Acoust Soc Am* 2003;114:1905–19.
- [19] Petach M, Tward E, Backhaus S. Design of a high efficiency power source (HEPS) based on thermoacoustic technology. Final report. NASA Contract No. NAS3-01103, CDRL, vol. 3f; 2004.
- [20] Backhaus S, Tward E, Petach M. Traveling-wave thermoacoustic electric generator. *Appl Phys Lett* 2004;85:1085.
- [21] Kang H, Zhou G, Li Q. Thermoacoustic effect of traveling-standing wave. *Cryogenics*; 2010.
- [22] Kang H, Zhou G, Li Q. Heat driven thermoacoustic cooler based on traveling-standing wave. *Energy Convers Manage* 2010;1–6.
- [23] Kang H, Li Q, Zhou G. Optimizing hydraulic radius and acoustic field of the thermoacoustic engine. *Cryogenics* 2009;49:112–9.
- [24] Hu ZJ, Li ZY, Li Q, Li Q. Evaluation of thermal efficiency and energy conversion of thermoacoustic Stirling engines. *Energy Convers Manage* 2010;51:802–12.
- [25] Babaei H, Siddiqui K. Design and optimization of thermoacoustic devices. *Energy Convers Manage* 2008;49:3585–98.
- [26] Yu Z, Jaworski AJ. Impact of acoustic impedance and flow resistance on the power output capacity of the regenerators in travelling-wave thermoacoustic engines. *Energy Convers Manage* 2010;51:350–9.
- [27] Rayleigh L. The explanation of certain acoustical phenomena. *Nature* 1878;18:319–21.
- [28] Kennedy J, Eberhart R. Particle swarm optimization. In: Proceedings of IEEE international conference on neural networks, vol. IV; 1995. p. 1942–8.
- [29] Birge B. Particle swarm optimization toolbox. MathWorks, optimization toolbox; 2006. <<http://www.mathworks.com/matlabcentral/fileexchange/7506>> [accessed 13.03.11].